

TWO HEURISTICS FOR IMPROVING THE EFFICIENCY OF A MARKOV CHAIN BASED DECISION MAKING METHOD

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ABSTRACT

The paper describes two heuristics to reduce the number of comparisons necessary to reach a certain goal for a Markov model for multi-criteria and multi-person decision making. The motivation results from a demand observed in the early stages of an innovation process. Here, many alternatives need to be evaluated by several decision makers with respect to several criteria. With the implementation of the heuristics the number of comparisons necessary could be decreased significantly. By reducing the evaluation effort necessary to reach a given goal, we will make the Markov-chain decision making method applicable to real world settings with a larger number of alternatives.

Keywords: multi-criteria decision making, reducing the number of necessary comparisons, heuristics

1. DESCRIPTION OF THE PROBLEM

We consider the problem of evaluating alternatives in the early stages of an innovation process. In this application area, alternatives need to be evaluated by several decision makers with respect to different criteria. There are possibly many alternatives that need to be considered; therefore it is necessary to make the evaluation process fast and simple. Consequently we address a multi-person and multi-criteria decision making problem (MPMCDM). The following example describes the intended application.

The early stages of a stage-gate process (Cooper 1988) often contain a large number of alternatives. In the very first stages all of them are described only with a title and a short characterisation. Because of limited resources an innovation team must identify only the top alternative to bring that forward to the next stage of the process.

Little or no quantifiable information is available about the alternatives in the first stages of a stage-gate process, therefore it is not possible to rank the alternatives based on objective criteria. Instead, only subjective impressions are available at this stage, enabling decisions of the form "A is better than B" with respect to a given criterion.

In Chelvier et al. (2008a) and Chelvier et al. (2008b) we described how to model the evaluation and

decision process in the given application as well as how to build a complete ranking.

In this paper we address the large number of necessary comparisons. This drawback is a well known issue of pairwise comparison-based methods and sometimes referred to as the information overload problem (Dryer 1990). Our solution is suitable if only the top alternative instead of the complete ranking is wanted. The following questions need to be addressed: How to identify the top alternative without building a complete ranking? How to reduce the number of necessary pairwise comparisons to reach this goal?

2. BACKGROUND

2.1. Multi-criteria decision making

Multi-criteria decision making (MCDM) is a discipline aimed at supporting decision makers who are faced with the evaluation of many alternatives with respect to several criteria (Roy 2000; Hokkanen 1997; Belton and Stewart 2002). Depending on which type of result is needed, many different MCDM methods are available. Thirty available methods are discussed, for example, in Guitouni, Martel and Vincke (1998).

In multi-person decision making (MPDM), more than one person is involved in the decision making process. Because most MCDM methods assume only one decision maker, strategies for mapping several opinions onto a single result are needed (Meixner and Haas 2002; Vetschera 1991; Eisenführ and Weber 1994).

In the decision making method used in this paper, the decision makers perform independent partial evaluations which are subsequently combined to obtain an overall set of evaluations, which form the basis of the ranking computation.

In the field of MCDM many methods have been developed for specialised applications. Seven methods which can be used in the early stages of an innovation process are AHP (the Analytic Hierarchy Process) (Saaty 1980), WISDOM (Van Overveld 2003), IPC (Incomplete Pairwise Comparisons algorithm) (Harker 1987), Incomplete AHP (Caklovic and Piskac 2001), cost-benefit analysis (CBA) (Chakravarty 1987), ELECTRE (Benayoun, Roy and Sussman 1966) and MPMCDM with Markov Chains (Chelvier et al. 2008a).

However, two methods are not directly applicable to the intended task. ELECTRE deals with soft evaluation criteria, but this method needs all pairwise comparisons to compute the results. The CBA method needs measurable and quantifiable criteria to compute a valid result.

Accordingly, ELECTRE and CBA cannot be the preferred methods to evaluate alternatives under the given constraints.

2.2. AHP and WISDOM methods

The Analytic Hierarchy Process is based on a hierarchy of evaluation criteria, and uses paired comparisons of alternatives with respect to these criteria. Gradations in the comparisons are expressed using numerical values. The ranking of the alternatives is obtained from an eigenvalue computation on a suitably aggregated matrix.

Known drawbacks of the AHP method are the need to deal with inconsistent sets of evaluations, the large number of pairwise comparisons needed, a complex mathematical model which is intransparent to the user and questionable rankings resulting from innocuous individual comparisons. The incomplete AHP method tries to use a smaller number of pairwise comparisons without compromising the result. IPC and Incomplete AHP are based on AHP and inherit these drawbacks.

The Weightless Incremental Selection and Ordering Method (WISDOM) is also based on paired comparisons with gradations. However, in this case, the gradations are qualitative, rather than quantitative. The evaluation criteria are also weighted using pairwise comparisons. An algebra for computations on these qualitative relations is then developed for which an iterative method is used to compute a ranking.

WISDOM shares a lot of the positive attributes of AHP. However, it also has several advantages over AHP. The first is the use of qualitative expressions rather than numerical ones for the pairwise comparisons and the computations. It is also able to explicitly detect evaluation inconsistencies. Its main disadvantage – as with AHP – is the lack of a natural extension to multi-user applications. One such extension, named TeamWISDOM, was presented by Weber (2007).

Our method is similar to both AHP and WISDOM, in that it is also based on pairwise comparisons of the alternatives with respect to different criteria and that it subsequently computes a ranking vector. Like AHP, our method also performs an eigenvalue computation on a matrix representing the results of the individual comparisons. However, in contrast to both methods, our approach does not provide for differing weights in the pairwise comparisons (although this extension would be easy to accommodate), and its overall structure is simpler.

2.3. Markov chains

Discrete-time Markov chains (DTMCs) are well researched mathematical models with many applications in Science and Engineering. A DTMC is

described by a stochastic matrix P and a probability vector π . The steady-state solution of the DTMC contains the probabilities of each of the system states and is given by the solution of the linear system of equations

$$\pi P = \pi \quad (1)$$

Markov chains are drawn as directed, annotated graphs, where the nodes represent the states and the arcs the possible state transitions. The weights associated with the arcs describe the one-step probabilities for each state transition. A state or set of states of a Markov chain is called absorbing, if it contains only incoming arcs.

We use the Markov chain-based decision making method, which was described in (Chelvier et al. 2008a and 2008b). Here, pairwise comparisons can be combined by weighting these according to the importance of the criteria and the decision makers, resulting in a discrete-time Markov chain. A random walk on this DTMC (Stewart 1994) models the decision process, where a longer state sojourn time implies a better alternative. The solution of the DTMC resulting from all pairwise comparisons then yields a complete ranking of all alternatives.

3. NEW IN OUR APPROACH

The Markov chain-based decision making method allows us to compute intermediate rankings during the evaluation process. Based on the intermediate rankings, we can use heuristics to estimate the effect of another comparison on the computed ranking. Comparing the effects of the possible comparisons, a heuristic suggests, which comparison should be done next, because it is expected to lead faster to the desired goal.

The goal is to evaluate the comparisons with biggest impact on the ranking first and the comparisons with lower impact on the ranking later. We assume that comparisons with high impact assign the right ranking position to the alternatives and comparisons with lower impact adjust the ranking position value but have little effect on the top ranking positions. Consequently, fewer comparisons are necessary to determine the top alternative.

4. HEURISTICS

We present two heuristics to estimate the next comparison with the highest impact on the intermediate ranking: *Maximum weight* and *Euclidean distance*.

4.1. Maximum Weight

The heuristic *Maximum weight* uses the information given by the initialisation of the evaluation. Therefore we denote the participants in the decision process by p_k with $k = 1 \dots K$ and the decision criteria by d_l , with $l = 1 \dots L$. We define a matrix A of dimension $K \times L$ whose coefficients α_{kl} satisfy $0 \leq \alpha_{kl} \leq 1$ and

$$\sum_{k=1}^{k=K} \sum_{l=1}^{l=L} \alpha_{kl} = 1 \quad (2)$$

Thus the coefficient α_{kl} contains information about the level of expertise of Participant p_k with respect to Criterion d_l as well as the degree of importance of Criterion d_l , where larger values imply greater importance. From all open comparisons the heuristic chooses the one with the largest combined weight of Criterion d_l and Evaluator p_k .

Consequently, the comparison order for each participant is known by the facilitator at the beginning of the evaluation process.

4.2. Euclidean distance

The *Euclidean distance* heuristic compares the possible effect of all open comparisons, choosing the one that can produce the ranking with the largest distance to the current ranking. It is, however, still a heuristic, since the actual value of the comparison is not known; if alternatives A and B are selected to be compared next, the evaluator still has the choice to select either one of them as superior. We assume that the comparison, which results in the maximum *Euclidean distance*, implies the biggest impact on the ranking.

Compared to the heuristic *Maximum weight* the comparison order is not known at the beginning of the evaluation process. Here, after each evaluation we estimate the next comparison with a “brute force” algorithm for all open comparisons.

With these heuristics the evaluation process can be stopped if the top alternative is identified and is not challenged for a number of comparisons. The better the heuristic the less pairwise comparisons are necessary.

5. ADVANTAGES IN OUR APPROACH

The heuristics reduce the number of necessary pairwise comparisons to obtain the desired evaluation result in the intended application. In that case the named information overload problem of the Markov chain-based decision making method can be reduced. We notice that the simpler the evaluation goal the higher the possible savings of pairwise comparisons.

The benefit of this approach is most significant if soft evaluation criteria are involved. The method works from two alternatives to more than one hundred alternatives. The limiting factor is the necessary number of pairwise comparisons to reach a certain goal.

6. EXPERIMENTS

In our intended application it is sufficient to identify the top alternative. To study the effect of the heuristics we choose a real MCDM problem from industry: In an ideation workshop many product ideas for an automotive supplier are generated. After the idea generation the participants of the workshop identified those alternatives with the highest expected benefit for the company. After the workshop the best alternative should be identified. Therefore the innovation process

provides evaluation criteria and roles of the decision maker. The initial situation is as follows:

- There are five decision makers as participants in the decision process: the General Manager, the Product Manager, the Marketing Manager, the Production Manager and the Sales Manager.
- There are nine new product alternatives as a result from the ideation workshop. Each alternative is described with a title, a short characterization and a list of pros and benefits.
- We have ten evaluation criteria: fitting to a megatrend, market potential, competitive situation, degree of innovation, strength of the unique selling proposition, potential sales volume, research and development costs, profit margin, proportion of value-added, proportion of strengths and weaknesses. In the following process we assume that all criteria are independent and we consider no feedback between the criteria.

Even though the criteria are not quantifiable with the given information, subjective impressions are available, enabling decisions of the form “A is better than B” with respect to a given criterion.

In preparation of our experiments we collected all comparisons from the decision makers in a personalised questionnaire. We asked each participant “You see the Alternatives m_1 and m_2 . Which of them is better with respect to the given criterion”? Each participant had to carry out 360 pairwise comparisons. The number T of all pairwise comparisons is given by

$$T = \frac{K * L * M * (M - 1)}{2} \quad (3)$$

where M is the number of alternatives and every participant makes every possible pairwise comparison with respect to every criterion.

In the next step we use a program to simulate four evaluation workshops, each with a different strategy for the order of the pairwise comparisons. The four workshops are:

1. Random order (reference 1)
2. Facilitator order (reference 2)
3. Maximum weight (heuristic)
4. Euclidean distance (heuristic)

In workshop 1 we use a randomised order of the evaluations. We assume that randomisation is representative for all workshops with no order strategy. Additionally, the program simulates a simultaneous treatment of the participants (all at the same time). We use the random order strategy as reference for the experiments with the heuristics.

In workshop 2 we use an order strategy with minimum cognitive load for the participants. This means that the comparisons are ordered first by criteria,

second by alternative 1 and third by alternative 2. Consequently the evaluation criterion and one alternative are changing as few as possible in the evaluation process. We assume that this strategy is representative for a good facilitated real workshop. As in workshop 1 the program simulates a simultaneous treatment of all participants.

In workshop 3 and 4 we use the presented heuristics to order the pairwise comparisons. We assume that the heuristics estimate the next comparison with the highest impact on the intermediate ranking. The evaluation simulated a simultaneous treatment of the participants as well.

7. RESULTS

The four diagrams show the probability values of the DTMC solution vector over 1800 pairwise comparisons, the lines also correspond to the ranking vector of the alternatives.

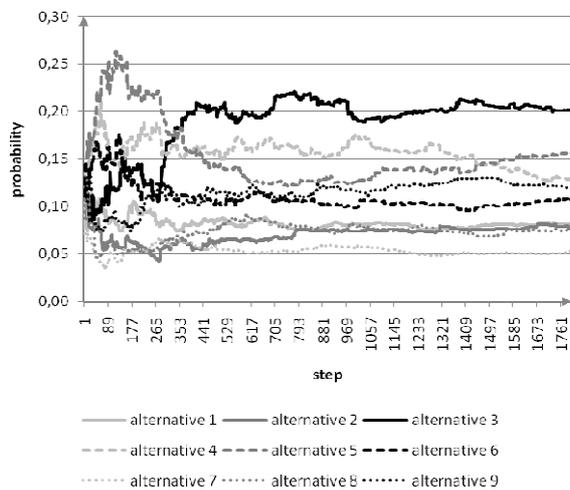


Figure 1: Order of Pairwise Comparisons is Randomised.

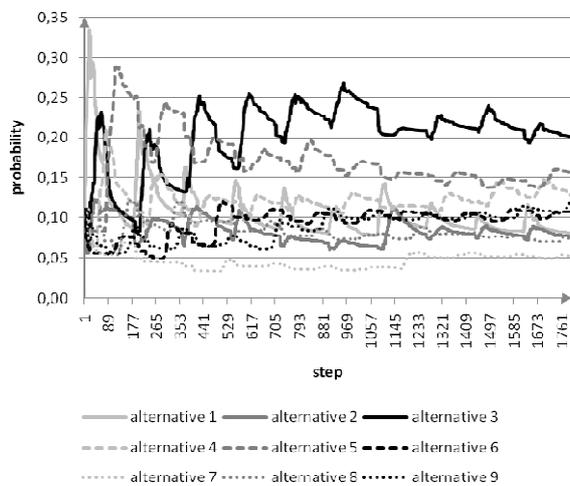


Figure 2: Order of Pairwise Comparisons for Minimum Cognitive Load (Facilitators order).

In Figure 1 the order of the pairwise comparisons is randomised. The best alternative is identified after approx. 350 comparisons. The second best alternative is identified after approx. 1530 comparisons. The late identification of the second best alternative shows the disadvantage of an arbitrary (in this case randomised) order of pairwise comparisons: Comparisons with a high impact might be done too late. One cannot be sure, when the evaluation process can be terminated even if only the top alternative is of interest.

In Figure 2 the best alternative is identified after approx. 560 comparisons. The second best alternative is identified after approx. 1700 comparisons. The late identification of the second best alternative is a disadvantage in that strategy, too. Additionally, the discontinuous progress of the values in the ranking vector makes the termination of the evaluation process very difficult even if only the top alternative is of interest.

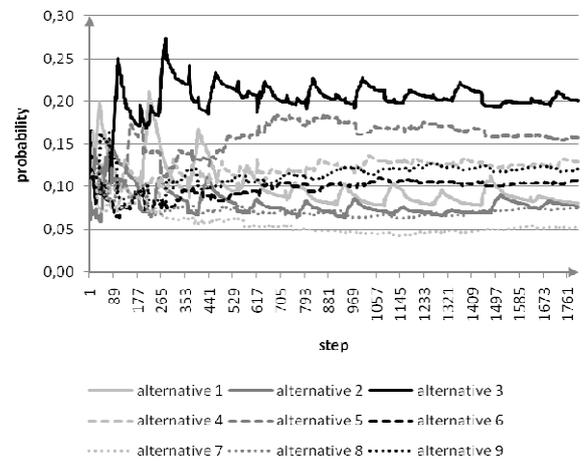


Figure 3: Order of Pairwise Comparisons is Computed with the Heuristic *Max Weight*.

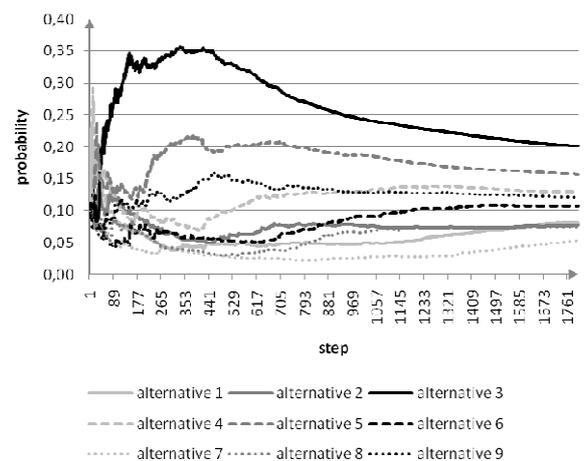


Figure 4: Order of Pairwise Comparisons is Computed with the Heuristic *Euclidean Distance*.

In Figure 3 the best alternative is identified after approx. 250 comparisons. The second best alternative is identified after approx. 450 comparisons. Later on,

ranking position swaps only occur in the lower ranking positions. Consequently the heuristic *Max weight* causes a positive effect towards the reduction of necessary comparisons in this example.

In Figure 4 the best alternative is identified after approx. 40 comparisons, the second best after approx. 200 comparisons. The heuristic *Euclidean distance* decreases the necessary comparisons significantly in this example. Another advantage of this heuristic is the smooth progress of the values in the ranking vector. Compared to the heuristic *Max weight*, trends might be more easily detectable.

In comparison to the reference strategies random order and facilitator order we can observe many improvements obtained from the heuristics:

1. The heuristic *Euclidean distance* avoids a ranking swap between the best and second best alternatives.
2. The heuristic *Euclidean distance* avoids the late ranking swap between the third best alternative and the fourth best alternative.
3. With the heuristics *Max weight* and *Euclidean distance* the best alternative and the second best alternative turned out very early.
4. The heuristic *Max weight* clearly identifies the best alternative and the second best alternative. Unfortunately, the progress of the values in the ranking vector is slightly discontinuous.
5. The heuristic *Euclidean distance* identifies the best alternative and the second best alternative earlier than heuristic *Max weight* and much earlier than both reference strategies.
6. The progress of the values with heuristic *Euclidean distance* is very smooth so that trends are appreciable.
7. In contrast to the strategies random order, facilitator order and heuristic *Max weight* the values in the ranking vector of heuristic *Euclidean distance* converge continuously to the final values.
8. The strategy random order identifies the best idea with less pairwise evaluations than the facilitator order. Nevertheless, the strategy random order cannot be more than a reference because one cannot be sure when the evaluation process can be terminated.
9. With the reference strategy facilitator order participants need less rethinking between pairwise comparisons (less cognitive load) than in the other strategies. Instead they need more comparisons to reach a certain evaluation goal. The promised benefit is annihilated.

Fortunately, in this example, the brute force algorithm to compute intermediate rankings in the heuristic *Euclidean distance* takes not more than one second between each comparison.

8. CONCLUSION

The goal of the paper was to reduce the necessary number of pairwise comparisons if only the best alternative is needed. We used two heuristics to choose the next comparison to be made, in order to identify the top alternative as early as possible in the evaluation process.

As the examples show, the presented heuristics could reduce the number of necessary comparisons significantly. In detail, the *Euclidean distance* heuristic shows better results than the *Max Weight* heuristic. We assume that the use of intermediate results in the *Euclidean distance* heuristic is one reason for the better performance.

In our future work we want to identify more heuristics and implement the algorithm in a group decision support system. By reducing the evaluation effort necessary to reach a given goal, we will make the Markov-chain decision making method applicable to real world settings with a larger number of alternatives.

Further work will also include developing a method for enforcing irreducibility which retains sparsity, the implementation of comparisons of the form "much better than", comparing the results with those obtained from other methods, determining the intersubjectivity of the computed ranking among the decision makers and studying the behaviour of the method using data from more real-life problems.

Another question we would answer in further work is the definition of a definite stop criterion whereby the evaluation process can be terminated.

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